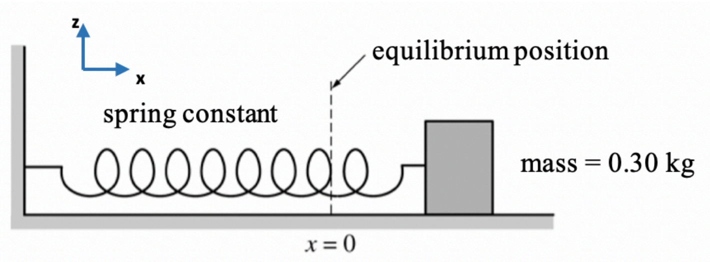
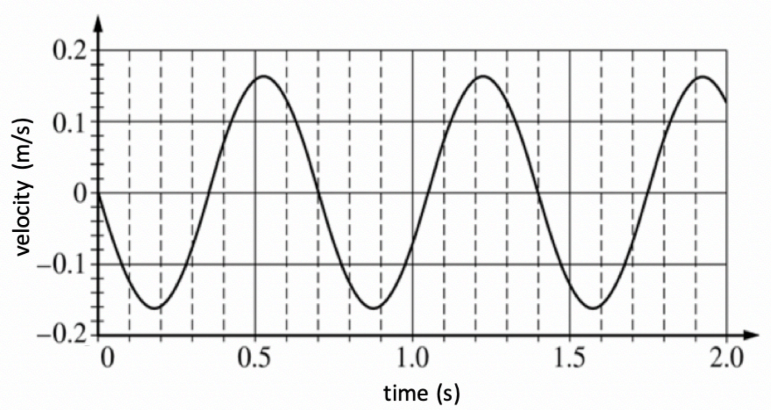
WRITTEN RESPONSES

1. (14 points) A horizontal spring, of spring constant k, is attached to a fixed wall on one end and a 0.30-kg block on the other end. Presume the table is frictionless.



After stretching the spring, the block is released from rest at time $t = 0$. A motion detector records the position of the block as it oscillates, producing the following velocity vs. time graph:



a. (4 points) Determine the equation for $v(t)$. Include numerical values for all constants.

criteria		points
Writes correct equation for $v(t)$, including negative sign.	$v(t) = -v_{max}\sin(\omega t) = -v_{max}\sin\left(\frac{2\pi t}{T}\right)$	1
Relates ω to frequency or period.	$\omega = 2\pi f$ or $\omega = 2\pi/T$	1
Reads the period from graph and solves for ω.	$\omega = \frac{2\pi}{T} = \frac{2\pi}{(0.70\text{ s})} = 9.0\ \frac{\text{rad}}{\text{s}}$	1
Reads the maximum speed from graph; substitutes all numerical values into $v(t)$.	$v(t) = -\left(0.16\ \frac{\text{m}}{\text{s}}\right)\sin\left(9.0\ \frac{\text{rad}}{\text{s}}t\right)$ (0.15 m/s < v_{max} < 0.17 m/s)	1

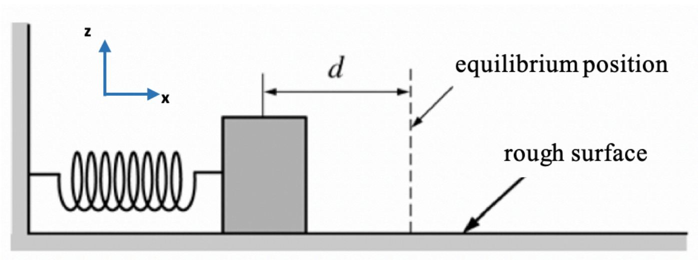
b. (2 points) Given the equilibrium position is at $x = 0$, determine the equation for $x(t)$. Include numerical values for all constants.

criteria		points
Sets up integral of $v(t)$ using answer from part a.	$x(t) = \int v(t)\,dt = -\int \left(0.16\ \frac{\text{m}}{\text{s}}\right)\sin\left(9.0\ \frac{\text{rad}}{\text{s}}t\right)dt$	1
Uses $v(t)$ from part a (or v_{max}/ω) to solve for x_{max}. Substitutes values for x_{max} and ω into $x(t)$.	$x_{max} = \left(0.16\ \frac{\text{m}}{\text{s}}\right)/\left(9.0\ \frac{\text{rad}}{\text{s}}\right) = 0.018\text{ m}$ $x(t) = (0.018\text{ m})\cos\left(9.0\ \frac{\text{rad}}{\text{s}}t\right)$	1

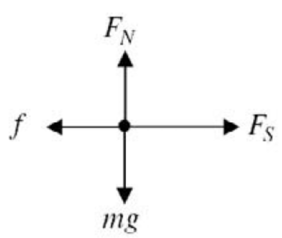
c. (2 points) Calculate the value of k.

criteria		points
Indicates relationship between T and k, or ω and k, or uses conservation of energy.	$T = 2\pi\sqrt{\frac{m}{k}}$ $\quad$ $\omega = \sqrt{\frac{k}{m}}$ $\quad$ $\frac{1}{2}kx_{\text{max}}{}^2 = \frac{1}{2}mv_{\text{max}}^2$	1
Substitutes correct values and solves for k.	$k = \frac{4\pi^2 m}{T^2}$ $\quad$ $k = \frac{4\pi^2(0.30\text{ kg})}{(0.70\text{ s})^2} = 24\ \frac{\text{N}}{\text{m}}$ $k = m\omega^2$ $\quad$ $k = (0.30\text{ kg})\left(9.0\ \frac{\text{rad}}{\text{s}}\right)^2 = 24\ \frac{\text{N}}{\text{m}}$ $k = \frac{mv_{\text{max}}^2}{x_{\text{max}}^2}$ $\quad$ $k = \frac{(0.30\text{ kg})\left(0.16\ \frac{\text{m}}{\text{s}}\right)^2}{(0.018\text{ m})^2} = 24\ \frac{\text{N}}{\text{m}}$	1

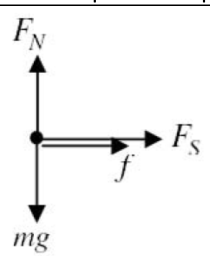
Now presume the block and spring are placed on a rough surface, as shown below. The block is displaced so that the spring is compressed a distance d and released from rest.



d. (3 points) Presume the spring is compressed a distance of $x = d/2$. Draw and label the forces that act on the block when it is moving (i) toward the equilibrium position, and (ii) away from the equilibrium position.



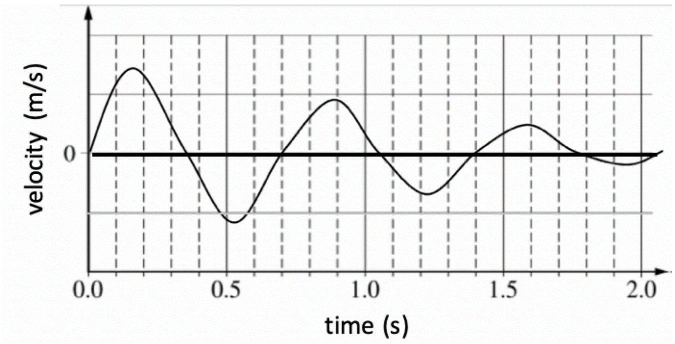
toward the equilibrium position



away from the equilibrium position

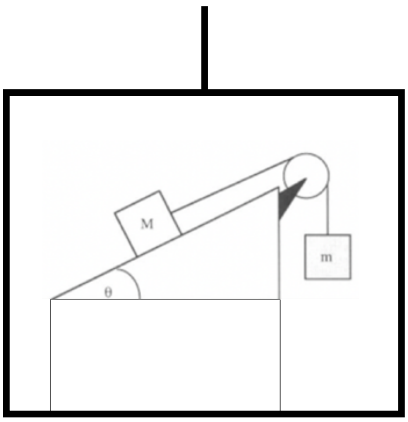
criteria		points
Gravitational and normal forces sketched equal in magnitude and opposite in direction.		1
Spring force sketched to the right.	Spring force sketched to the right.	1
Friction force sketched to the ***left*** and smaller in magnitude than the spring force.	Friction force sketched to the ***right*** and smaller in magnitude than the spring force.	1
A maximum of 1 point was deducted for any extra forces listed.		

e. (3 points) Sketch a velocity vs. time graph for this system **for at least three cycles**. Assume there is negligible change in the period.

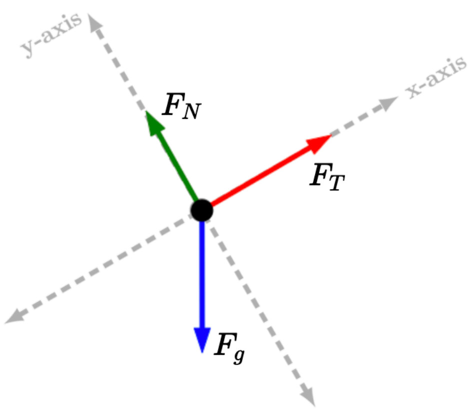
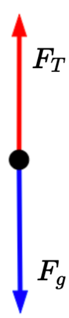


criteria	points
Both axes are labelled and scale is included for the x-axis.	0.5
Graph starts at zero with increasing positive velocity.	1
Graph displays damped oscillations over time.	1
Graph passes through equilibrium at 0.35 s intervals.	0.5

2. (14 points) You are inside an elevator where two blocks are connected by a massless non-elastic string over an ideal pulley; the elevator is at rest on the ground floor. One block (mass M) is on a frictionless incline with angle θ while the other block (mass m) hangs over the pulley.



a. (2 point) Sketch and label the free body diagrams for each block.

criteria			points
Force(s) sketched correctly. Gravitational and tension forces for mass m sketched equal in magnitude and opposite in direction.	mass M 	mass m 	2
A maximum of 1 point was deducted for any extra forces listed.			

b. (3 points) Presume that $M = 5$ kg, $m = 2$ kg, and $\theta = 15°$. Determine the acceleration $\vec{a}$ of the two masses when mass m is released; show all calculations.

criteria		points
Uses Newton's Second Law to address the forces acting on mass M.	$\sum \vec{F}_{\text{M,x}} = \vec{F}_{\text{T}} - M\lvert\vec{g}\rvert \sin\theta = M\vec{a}$	1
Uses Newton's Second Law to address the forces acting on mass m.	$\sum \vec{F}_{\text{m,y}} = m\lvert\vec{g}\rvert - \vec{F}_{\text{T}} = m\vec{a}$	1
Substitutes correct values and solves for $\vec{a}$. Positive acceleration means the direction is to the right/downward. (Any form of magnitude and direction will be accepted in the final answer.)	$\vec{F}_{\text{T}} - M\lvert\vec{g}\rvert \sin\theta + m\lvert\vec{g}\rvert - \vec{F}_{\text{T}} = m\vec{a} + M\vec{a}$ $\vec{a} = \frac{\lvert\vec{g}\rvert(m - M\sin\theta)}{(m + M)}$ $\vec{a} = \left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{2\text{ kg} - 5\text{ kg}\sin(15°)}{2\text{ kg} + 5\text{ kg}}\right) = +0.99\ \frac{\text{m}}{\text{s}^2}$	1

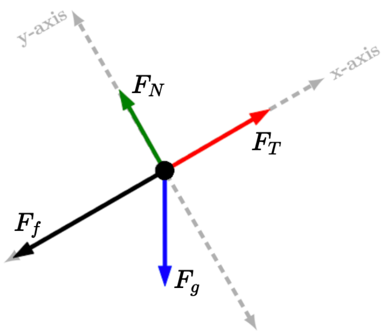
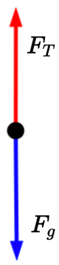
c. (4 points) You step outside the elevator. The elevator now **accelerates upward with constant acceleration $(1/6)g$**. From your perspective outside of the elevator, what is the acceleration $\vec{a}$ of each block? Show all calculations.

criteria		points
Inside the elevator Addresses increase in $\vec{g}$ due to the acceleration of the elevator with respect to the ground $\lvert\vec{a}_{\text{elevator}}\rvert$.	$g' = \lvert\vec{g}\rvert + \lvert\vec{a}_{\text{elevator}}\rvert$ $\qquad g' = \frac{7}{6}\lvert\vec{g}\rvert$	0.5
Inside the elevator (both masses) Determines acceleration of system, which is the same for both masses. Positive acceleration means the direction is to the right/downward.	$\vec{a}_{\text{in,system}} = g'\frac{(m - M\sin\theta)}{(m + M)}$ $\vec{a}_{\text{in,system}} = \left(\frac{7}{6}\lvert\vec{g}\rvert\right)\left(\frac{2\text{ kg} - 5\text{ kg}\sin(15°)}{7\text{ kg}}\right) = +1.15\ \frac{\text{m}}{\text{s}^2}$	1
Outside the elevator (mass m) Determines acceleration of mass m, noting that $\vec{a}_{\text{elevator}}$ and $\vec{a}_{\text{in}}$ are opposite in direction. Positive acceleration means the direction is downward.	$\vec{a}_{\text{out},m} = \vec{a}_{\text{elevator}} + \vec{a}_{\text{in,system}}$ $\vec{a}_{\text{out},m} = \left(\frac{\lvert\vec{g}\rvert}{6}\right) - \left(1.15\ \frac{\text{m}}{\text{s}^2}\right) = +0.48\ \frac{\text{m}}{\text{s}^2}$	1
Outside the elevator (mass M) Uses vector addition (law of cosines) to determine acceleration of mass M. Positive acceleration means the direction is to the right.	$\vec{a}_{\text{out},M} = \vec{a}_{\text{in,system}} + \vec{a}_{\text{elevator}}$ $\vec{a}_{\text{out},M} = \sqrt{\lvert\vec{a}_{\text{in}}\rvert^2 + \lvert\vec{a}_{\text{elevator}}\rvert^2 - 2\lvert\vec{a}_{\text{in}}\rvert\lvert\vec{a}_{\text{elevator}}\rvert\cos(105°)}$ $\vec{a}_{\text{out,mass }M} = +2.22\ \frac{\text{m}}{\text{s}^2}$	1.5

d. (1 point) Explain the significance of your answers from part c when compared to your answers from part b.

Indicates magnitude of the acceleration for each mass from part b is the same while the magnitude of the acceleration for each mass from part c is different.	0.5
Identifies the acceleration of the blocks-pulley system as the cause for the different accelerations.	0.5

e. (4 points) **The elevator now stops at the top floor and stays there**. The surface of the incline is replaced with a different material such that the coefficients of friction between the block and the incline are $\mu_s = 0.3$ and $\mu_k = 0.15$. Will the masses move when mass m is released from rest? Explain your response, including free-body diagrams, for full credit.

<table>
<tr><th colspan="3">criteria</th><th>points</th></tr>
<tr><td>Calculates net force to determine direction of system. Positive net force means the direction is to the right/downward.</td><td colspan="2">$$\vec{F}_{\text{net}} = mg - M|\vec{g}|\sin\theta$$ $$\vec{F}_{\text{net}} = (2\text{ kg})\left(9.8\frac{\text{N}}{\text{kg}}\right) - (5\text{ kg})\left(9.8\frac{\text{N}}{\text{kg}}\right)\sin(15°) = +6.9\text{ N}$$</td><td>1</td></tr>
<tr><td>Determines the magnitude of the static frictional force for mass M.</td><td colspan="2">$$|\vec{F}_{SF}| = \mu_S\vec{F}_N = \mu_s M|\vec{g}|\cos\theta$$ $$|\vec{F}_{SF}| = (0.30)(5\text{ kg})\left(9.8\frac{\text{N}}{\text{kg}}\right)\cos(15°) = 14.2\text{ N}$$</td><td>1</td></tr>
<tr><td>Forces are sketched correctly. Since the net acceleration of the system is to the right for mass M and less than the magnitude of the static frictional force, the static frictional force points towards the left and is greater in magnitude than the force due to tension.</td><td>mass M

</td><td>mass m

</td><td>1</td></tr>
<tr><td colspan="3">Indicates that since the magnitude of the net force on the system is less than the magnitude of the static frictional force, both masses in the system will not move.</td><td>1</td></tr>
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